Review of Error Analysis and Practice Problems for

PHY 201L/211L

and

PHY 202L/212L

General College/University Physics Lab

Barry University

Physical Sciences Department

Contents

1 Error Analysis

1.1 The Meaning of Error in Science

In science, the term error does not carry the negative connotation of the term mistake. Every scientific measurement is subject to errors, and the role of the scientist is to understand and quantify them (*since you cannot avoid them, learn how to deal with them*).

Let us start with a discussion of some of the possible sources of errors in a scientific experiment:

- **Precision**: This term refers to how fine the measurement scale is. For example, a ruler which reports millimeters is more precise that one which reports only centimeters. However, no instrument we use for measuring is perfect. As a thumb rule, the precision of an instrument is taken to be half the minimal increment that the instrument can measure. For example, is a scale reports grams as its minimal unit, the precision of that scale is taken to be 0.5 grams.
- **Random Errors**: These types of errors produce measurements that are randomly a little higher or a little lower than the true value of the quantity we are measuring. There are different sources of random errors. An example is the *measurement error* refers to our ability to perform the measurement. This can mean, for example, our ability to stop a watch at the right time. On the other hand, the *intrinsic random uncertainty* refers to random sources of error which are not connected with our ability, but are due to uncontrollable physical effects such as thermal or electromagnetic fluctuations, random noise, etc. In precision measurements, also quantum fluctuations could be a source for errors. If these fluctuations are random, then they are also considered random errors.
- **Systematic Errors**: These kinds of errors are due to non-random effects which produce an error in the measurement. For example, a slow watch would measure the wrong time even if we were very careful. Another example could be the use of the wrong value for a parameter needed in the measurement.

Another difficulty associated with a measurement is the so called **problem of definition** [see, e.g., Taylor (1997)[1]]. Suppose we want to measure a rectangular piece of wood. The size of the piece changes with temperature, humidity, etc. So it is important to specify what we mean by size (size at what temperature and humidity?) if the measurement aims to be accurate enough to be sensitive to those temperature and humidity conditions.

1.2 Importance in Understanding Errors

In some cases it does not seem necessary to understand the errors very much. When we plan a car trip we are not interested in knowing the distance to the accuracy of 1 foot.

In certain cases, however, it is very important to understand the uncertainty associated with a measurement. Suppose, for example, that a police officer has a very rudimentary instrument to detect the velocity of cars, which is accurate at the level of 10 miles per hour. Suppose that on a street the speed limit is 35 miles per hour and he stops a car which, according to his instrument, is going 40 miles/hour. Because of the imprecision of his instrument, the actual car velocity is somewhere between 30 and 50 miles/hour, so the officer cannot really give a ticket to the driver (if he knows the error associated with this measurement). If, instead, his instrument has an accuracy of

[1] John Taylor, An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, University Science Books; 2nd edition (March 1997).

1 mile/hour, then the agent could conclude that the car was speeding since its velocity is somewhere between 39 and 41 miles/hour.

1.3 Reporting Errors

The standard way to present the result of a measurement in a scientific report is

$$x \pm \Delta x\,, \qquad \text{or} \quad x \pm \delta x\,, \tag{1.1}$$

which means *Measured Value* ± *Uncertainty.* Notice that both notations, Δ and δ, are used in the literature, and in this manual we will also use both symbols to indicate the uncertainty[2].

The uncertainty can be associated with the imperfection of the measuring instrument or with other effects, as explained above. Ideally, it would be useful to explain the source of uncertainty in the experiment report.

When we write the result of our measure in the form above, Δx should be rounded to one significant figure, except when the leading digit in the uncertainty is 1. In fact, it does not make sense to say that the uncertainty is, for example, 5.145 since 0.145 is a small correction to the leading digit 5. However, if, for example, we find $\Delta x = 1.4$, it would be a good idea to report the second digit, since 0.4 is not really negligible with respect to 1. In addition, the last significant figure in x should be of the same order of magnitude (in the same decimal position) as the uncertainty. For example, 11.3 ± 5 does not make much sense. In fact, since the error is 5, it is not useful to report also the decimal figure in our result. The correct form is 11 ± 5. Other correct examples are: 110 ± 30, 11.3 ± 0.2, and 1905 ± 2.

Problem: Which of the following are correct ways to report the value of a measurement of a length?

a. (110 ± 10) m
b. (110 ± 11.2) m
c. (110 ± 1) m
d. (8.33 ± 1) m
e. (833.765 ± 0.005) m

[2]The symbol Δ (read *delta*) indicates a capital D in the Greek alphabet, and δ indicates a small d.

2 Propagation of Errors

In experiments, sometimes we can measure certain quantities directly. More often, however, we have to use relations to find the result we want. Suppose, for example, that we want to measure the volume of a rectangular box. We do not measure the volume directly. We have to measure the different edges first, and then we need to multiply the results to find the volume. In this case, we need to understand how to quantify the error associated with the quantity we wanted to measure in the first place (in our example the volume). This process is known as *error propagation* and indicates that, for example, the uncertainties associated with the measurements of the three edges *propagate* in an uncertainty in the value of the volume.

2.1 Error of the Sum and Difference of two Measured Quantities

Suppose we measure the two quantities X and Y and find $X = x \pm \Delta x$ and $Y = y \pm \Delta y$. We want to calculate the uncertainty associated with $S = X + Y$. Obviously, S can be as big as $x + \Delta x + y + \Delta y$, and as small as $x - \Delta x + y - \Delta y$. So

$$(x + y) + (\Delta x + \Delta y) < S < (x + y) - (\Delta x + \Delta y)$$

and therefore the uncertainty in S is $\Delta S = \Delta x + \Delta y$.

It is easy to show that the same uncertainty applies to the difference $D = X - Y$ of X and Y. In fact, D can be as big as $(x - y) + (\Delta x + \Delta y)$, and as small as $(x - y) - (\Delta x + \Delta y)$. So, the uncertainty in D is $\Delta D = \Delta x + \Delta y$. To recapitulate,

$$\Delta(x \pm y) = \Delta x + \Delta y\,. \tag{2.1}$$

Note that the error associated with the difference of two measurements is the sum, not the difference, of the errors associated with each one. Just remember that errors always add up.

2.2 Error in the Product of two Measured Quantities

Here we want to calculate the uncertainty in the quantity $P = xy$, knowing that the uncertainty associated with x is Δx, and that associated with y is Δy. To calculate the uncertainty in P, notice that P can be as big as $(x + \Delta x)(y + \Delta y) = xy + x\Delta y + y\Delta x + \Delta x\Delta y$, and as small as $(x - \Delta x)(y - \Delta y) = xy - x\Delta y - y\Delta x + \Delta x\Delta y$. Now, obviously, we expect that $\Delta x \ll x$ and $\Delta y \ll y$. In this case the last term, $\Delta x\Delta y$, can be neglected. So, we find

$$(xy) - (x\Delta y + y\Delta x) < P < (xy) + (x\Delta y + y\Delta x)$$

Therefore,

$$\Delta P = y\Delta x + x\Delta y$$

Finally, dividing by xy, we find

$$\frac{\Delta P}{P} = \frac{\Delta x}{x} + \frac{\Delta y}{y} \tag{2.2}$$

2.3 Error in the power of a Measured Quantity

Suppose we want to calculate the uncertainty in the quantity $P = x^n$, knowing that the uncertainty associated with x is Δx. To do that, let's write x^n as the product of x with itself for n times. Since we know how errors propagate in the product, we can calculate the error in P as

$$\frac{\Delta P}{P} = \frac{\Delta x}{x} + \frac{\Delta x}{x} + \frac{\Delta x}{x} + ..., (n \text{ times}) = \frac{n\Delta x}{x}$$

An interesting fact is that this formula is always valid, even if n is not positive or not integer. In general we have that, if $P = x^n$,

$$\frac{\Delta P}{P} = \frac{|n|\Delta x}{x} \tag{2.3}$$

where $|n|$ is the absolute value of n.

Problem: Use this formula to find the error associated with the square root of a quantity.

Solution: $\Delta\left(\sqrt{x}\right) = \Delta\left(x^{1/2}\right) = \frac{1}{2}\sqrt{x}\left(\frac{\Delta x}{x}\right)$

2.4 Error of the Quotient of two Measured Quantities

Consider, finally, the quantity $Q = x/y$, where the uncertainty in x is Δx and in y is Δy. We can calculate the uncertainty associated with Q, by writing $Q = xy^{-1}$, and using the previous results for the uncertainty associated with the product and with the power: Dividing x by y we find:

$$\frac{\Delta(xy^{-1})}{xy^{-1}} = \frac{\Delta x}{x} + |-1|\frac{\Delta y}{xy}$$

which means

$$\frac{\Delta Q}{Q} = \frac{\Delta x}{x} + \frac{\Delta y}{y} \tag{2.4}$$

2.5 Errors in Composed Expressions

Some expressions are combinations of sums or a products. In this case we can use the *chain rule* to find how errors propagate. For example, suppose we want to calculate the uncertainty in the expression

$$x^3 + y^2\,, \tag{2.5}$$

knowing that the uncertainty in x is δx, and the uncertainty in y is δy.

First of all, we note that the expression indicates a sum of x^3 and y^2. So, the first step is

$$\delta(x^3 + y^2) = \delta(x^3) + \delta(y^2)\,. \tag{2.6}$$

Now, each of the expressions represents a power. We can expand each of them and get

$$\delta(x^3) + \delta(y^2) = x^3\left(3\frac{\delta x}{x}\right) + y^2\left(2\frac{\delta y}{y}\right) = 3x^2\,\delta x + 2y\,\delta y\,. \tag{2.7}$$

So, the result is

$$\delta(x^3 + y^2) = 3x^2\,\delta x + 2y\,\delta y\,. \tag{2.8}$$

3 Graphics

3.1 Graphical Analysis

- Ask the Administrative Assistant in Wiegand 121 to log you in BEFORE you start using the Graphical Analysis program so that you may be able to print from that room's printer- otherwise, you will have to save your work on a disk).

1. Double click on "GA-Graphical Analysis 3.0" icon. (A data table and blank graph will appear.)

2. On the data table, double-click on the x (you may also double click on the x of the x-axis on the graph).

   (a) Under the **Column Definition** tab, type the name that will appear on the x-axis and write the units (if any).

   (b) Under the **Options** tab, under **Displayed Precision**, indicate to how many decimal places or significant figures your data for the x-axis should contain.

3. On the data table, double-click on the y (you may also double-click on the y of the y-axis on the graph).

   (a) Under the Column Definition tab, type the name that will appear on the y-axis and write the units (if any).

   (b) Under the **Options** tab, under **Displayed Precision**, indicate to how many decimal places or significant figures your data for the y-axis should contain.

4. On the toolbar click on **Analyze** and then click on **Linear Fit**. (Automatically, a box will appear on the graph pointing to the best fitting line with the $y = mx + b$ equation, the value for the slope (m), and the value for the y-intercept (b).)

   or..

   Click on **Analyze** and then click on **Curve Fit**. Choose which general equation your data points should fit (represent). Click on **Try Fit** and then click **ok**.

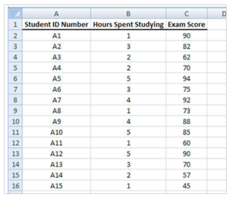
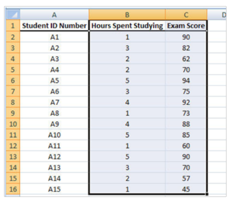
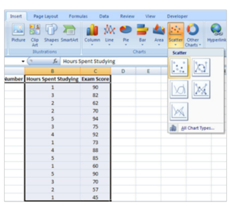
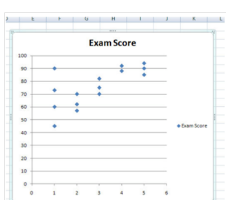
If adding a second data set

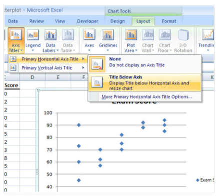
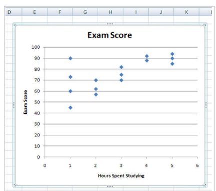
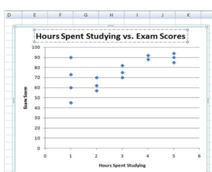
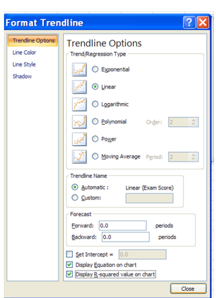
- Select "New Data Set" from the data menu. A new data table should appear.
- Follow steps 1-3 as indicated above.
- Click on the y on the y-axis of the graph and check the box for each data set that is to appear on the graph.

Note: For further information, there is a Graphical Analysis Manual in Wiegand 148 (The Manual has a pink cover).

3.2 Microsoft Excel

(adapted from www.brighthub.com)

<table>
<tr><td>
<table>
<tr><th>Student ID Number</th><th>Hours Spent Studying</th><th>Exam Score</th></tr>
<tr><td>A1</td><td>1</td><td>90</td></tr>
<tr><td>A2</td><td>3</td><td>82</td></tr>
<tr><td>A3</td><td>2</td><td>62</td></tr>
<tr><td>A4</td><td>2</td><td>70</td></tr>
<tr><td>A5</td><td>5</td><td>94</td></tr>
<tr><td>A6</td><td>3</td><td>75</td></tr>
<tr><td>A7</td><td>4</td><td>92</td></tr>
<tr><td>A8</td><td>1</td><td>73</td></tr>
<tr><td>A9</td><td>4</td><td>88</td></tr>
<tr><td>A10</td><td>5</td><td>85</td></tr>
<tr><td>A11</td><td>1</td><td>60</td></tr>
<tr><td>A12</td><td>5</td><td>90</td></tr>
<tr><td>A13</td><td>3</td><td>70</td></tr>
<tr><td>A14</td><td>2</td><td>57</td></tr>
<tr><td>A15</td><td>1</td><td>45</td></tr>
</table></td><td>Step 1: Enter or copy/paste your data into an Excel worksheet. As an example in this tutorial, we'll be using data consisting of hours spent studying and final exam scores for a select group of students.</td></tr>
<tr><td></td><td>Step 2: Highlight the columns that contain the data you want to represent in the scatter plot. In this example, those columns are Hours Spent Studying and Exam Score.</td></tr>
<tr><td></td><td>Step 3: Open the Insert tab on the Excel ribbon. Click on Scatter in the Charts section to expand the chart options box. Select the first item, Scatter with only Markers, from this box. After making this selection, the initial scatter plot will be created in the same worksheet. You can resize this chart window and drag it to any other part of the worksheet.</td></tr>
<tr><td>
</td><td>Step 4: Make any formatting or design changes you wish in the Design, Layout, and Format tabs located under Chart Tools on the Excel ribbon. For example, every graph need axes labels with units, and a title.</td></tr>
</table>

	**Label the Axes**: Horizontal axis: select the Layout tab under Chart Tools. Next, click on Axis Titles in the Labels section. Choose Primary Horizontal Axis and then pick Title Below Axis. A text box with the default wording Axis Title will appear on the chart. Click anywhere in that text box and edit the information to reflect the true title of the horizontal axis. Similarly, you can create a label for the vertical axis, but you will have more choices for title placement here. You'll need to use the Rotated Title option.
	**Chart Legend**: The default legend that was created with the scatter plot serves no real purpose here, so you can get rid of it. Go back to the Layout tab and click on Legend. From the list of expanded options, pick None to turn off the legend. Now your chart should now look like the one in the screenshot on the right.
	**Change Chart Title**: Click on the title to open the text box that contains it and edit it with your new description.
	**Add a Trendline**: Adding a trendline asks the computer to draw the best line of fit through your experimental data. You can include the equation for that line, which will have the general form $y = mx + b$. Right-click on any point on your graph, and select **Add Trendline** from the menu that appears. Select **Linear** and make sure the boxes **Display Equation on chart** and **Display R-squared value on chart** are selected. If there is a point in your data that must have a value of 0 for both variables (x and y), you can place a check mark next to Set Intercept = 0 to make your fit more precise. When you click close, these things will appear on your chart inside a text box. You can move the textbox anywhere you like.

3.3 Error associated with the slope

To find the error associated with the slope using Microsoft Excel, you should include the package " Data Analysis". To do that:

1. Click on the Microsoft Office icon on the top left corner. At the bottom of that drop down menu, click the icon Excel Options.
2. On the left side, click on Add-Ins and at the bottom it should by default say "Manage: Excel Add-Ins," press Go and place a check mark next to Analysis TookPak and press OK (if it asks you to install it, click Yes)
3. After installing the Analysis TookPak, on the top of the page click the Data tab, then on the right side on the "Analysis" section below the tabs there should be an icon that says Data Analysis, click on it.
4. Go down the list until you find Regression. Click to highlight it and press OK.
5. Select the data and keep in mind whether zero was a constant in your experiment.
6. Press OK.

4 General Problems on the Calculation of the Uncertainties

4.1 Proper rounding and proper units

1. You measure the length x of an objects. Which of the following ways to report the result are correct?

   (a) $x = (5.232 \pm 0.001)$

   (b) $x = (5.2 \pm 0.001)$ mm

   (c) $x = (5.232 \pm 0.001)$ mm

   (d) $x = (5.232 \pm 0.1)$ mm

   (e) $x = (5 \pm 1)$ mm

4.2 Propagation of errors

2. The two quantities x and y have an uncertainty x and y. Calculate the error in the following cases:

   (a) $x - 2y$

   Answer: $\delta x + 2\delta y$

   (b) $4x - 5y$

   (c) $3xy$

   Answer: $3xy\left(\frac{\delta x}{x} + \frac{\delta y}{y}\right)$

   (d) $\frac{x^3}{y^5}$

   Answer: $\frac{x^3}{y^5}\left(3\frac{\delta x}{x} + 5\frac{\delta y}{y}\right)$

   (e) $\sqrt{x}$

   Answer: $\frac{1}{2}\sqrt{x}\,\left(\frac{\delta x}{x}\right)$

   (f) $\sqrt{x^3}$

   (g) $\sqrt{2xy}$

   (h) $2\sqrt{\frac{x}{3y}}$

   Answer: $\sqrt{\frac{x}{3y}}\left(\frac{\delta x}{x} + \frac{\delta y}{y}\right)$

   (i) $2\sqrt{\frac{x^3}{3y}}$

(j) $\frac{x}{y} + \frac{y}{x}$

Answer: $\delta\left(\frac{x}{y}\right) + \delta\left(\frac{y}{x}\right) = \frac{x}{y}\left(\frac{\delta x}{x} + \frac{\delta y}{y}\right) + \frac{y}{x}\left(\frac{\delta x}{x} + \frac{\delta y}{y}\right) = \left(\frac{x}{y} + \frac{y}{x}\right)\left(\frac{\delta x}{x} + \frac{\delta y}{y}\right)$

(k) $x^2 + y^2$

Answer: $\delta x^2 + \delta y^2 = x^2\left(2\frac{\delta x}{x}\right) + y^2\left(2\frac{\delta y}{y}\right) = 2x\,\delta x + 2y\,\delta y$

4.3 Other problems on propagation of errors

3. You measure two interval of time with a stop watch which has a precision equal to 0.01s and find $t_1 = (35.23 \pm 0.01)$s, and $t_2 = (15.71 \pm 0.01)$s. What is $t_1 - t_2$ (including the error)?

4. You measure a quantity x and find $x = (3.2 \pm 0.1)$m/s. Calculate the value of q (including the error), where q is given by:
$$q = \frac{1}{x}$$

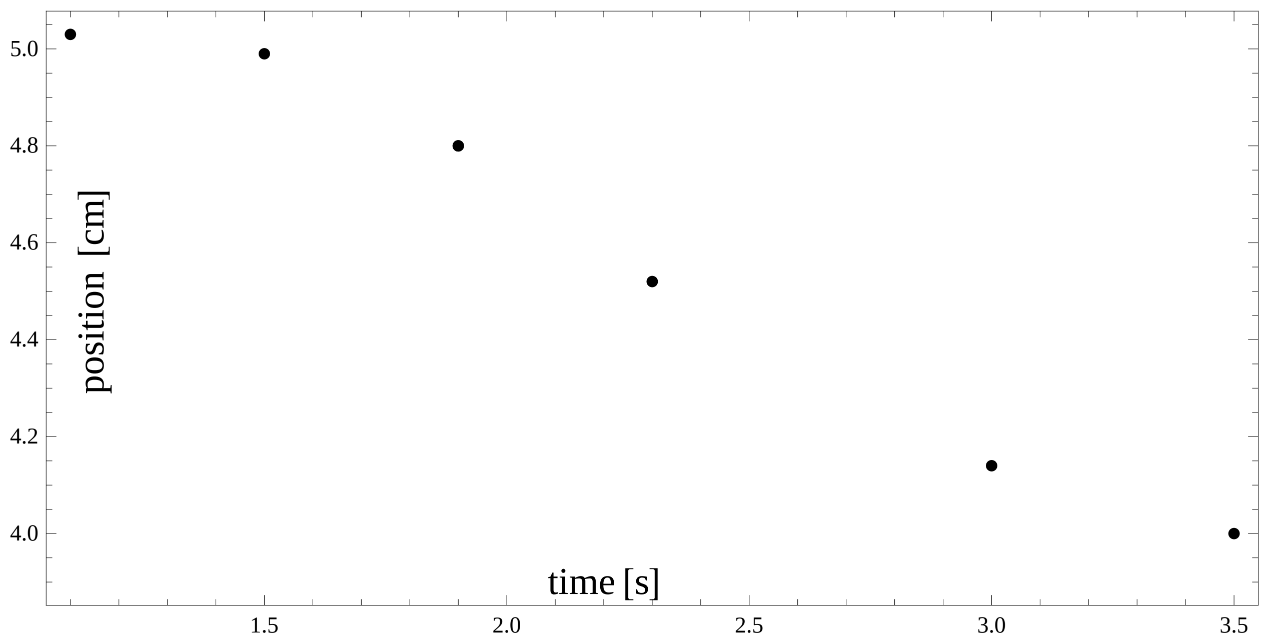
5. You measure the radius of a circle and find $R = (22.2 \pm 0.1)$cm. What is the area of the circle (including the error)?

6. You measure the radius of a sphere and find $R = (22.2 \pm 0.1)$cm. What is the volume of the sphere (including the error)?

7. The radius of a sphere is $R = (22.2 \pm 0.1)$cm. The radius of the base of a cylinder is $r = (12.0 \pm 1.2)$cm, and its height is $h = (24.4 \pm 1.1)$cm. What is the total volume occupied by the sphere and by the cylinder (including the error)?

5 Problems On Graphing

1. (**Points: 5**) The data shown in the figure below were taken in an experiment which measured the velocity of an object. Draw the best fitting line by eye (without using the LSA formula) and write the corresponding velocity of the object. Use SI units for the velocity.



Answer $v = -0.005$ m/s

2. In an experiment to find the acceleration due to gravity, g, you measure the position (in cm) of a free falling object at several different times (in seconds). When you plot your data (position in cm on the y-axis and time in s on the x-axis) in excel (or graphical analysis), the program gives for the best fit curve

$$y = 422.12\,x^2 + 5.92\,x + 2.04 \tag{5.1}$$

   (a) What is your result for the "acceleration due to gravity", g? Use SI units for your answer.

   (b) Suppose that the coefficients in (6.5) have a 5% error. What is your result for the "acceleration due to gravity", g including the error? Use SI units for your answer.

3. The force exert on a wire long L and with a current I by a magnetic field perpendicular to the wire is $|\vec{F}| = IL|\vec{B}|$. An experimental plot shows $|\vec{F}|$ as a function of L. The plot is a straight line, with equation (in SI units)

$$y = (11 \pm 1) \times 10^{-5} x + (0.021 \pm 0.028). \tag{5.2}$$

   The current in the wire is $I = (16 \pm 2)$mA. Find $|\vec{B}|$, including the error (the SI units for B is tesla, T).

4. Suppose that the two variables p and q are connected in the following way:

$$p^2 = \frac{3\pi}{\alpha}\, q\,.$$

If you want to find α from an experiment, you need to make a linear plot using several values for p and q, and then find α from the slope of that graph. Explain the way you would do it:

Plot .... on the y-axis and .... on the x-axis.

Find α as: ....

and the uncertainty in α as: ....

Solution: One possibility is to plot p^2 on the $y-$axis and q on the $x-$axis. The result would be a straight line with slope $m = 3\pi/\alpha$ and intercept $b = 0$. Therefore,

$$\alpha = \frac{3\pi}{m}\,, \qquad \delta\alpha = \frac{3\pi}{m}\left(\frac{\delta m}{m}\right) = \alpha\left(\frac{\delta m}{m}\right) \tag{5.3}$$

5. The same problem as before, but with:

$$pq = -\sqrt{3\alpha}$$

Plot .... on the y-axis and .... on the x-axis.

Find α as:

and the uncertainty in α as: ....

6. The same problem as before, but with:

$$p = -\pi\sqrt{\frac{\alpha}{q}}$$

Plot .... on the y-axis and .... on the x-axis.

Find α as:

and the uncertainty in α as:

7. Suppose that the two variables p and q are connected in the following way:

$$p^2 = \frac{3\pi}{\alpha}\, q + \frac{3}{2\pi h^2}\,.$$

If you want to find α and h from an experiment, you need to make a linear plot using several values for p and q, and then find α from the slope of that graph. Explain the way you would do it:

Plot .... on the y-axis and .... on the x-axis.

Find α as: .... and the uncertainty in α as: ....

Find h as: .... and the uncertainty in h as: ....

Solution: If you plot p^2 on the $y-$axis and q on the $x-$axis, the result would be a straight line with slope $m = 3\pi/\alpha$ and intercept $b = 3/(2\pi h^2)$. Therefore,

$$\alpha = \frac{3\pi}{m}\,, \qquad \delta\alpha = \frac{3\pi}{m}\left(\frac{\delta m}{m}\right) = \alpha\left(\frac{\delta m}{m}\right) \tag{5.4}$$

and

$$h = \sqrt{\frac{3}{2\pi b}}\,, \qquad \delta h = \sqrt{\frac{3}{2\pi b}}\left(\frac{1}{2}\frac{\delta b}{b}\right) = h\left(\frac{1}{2}\frac{\delta b}{b}\right) \tag{5.5}$$

8. The same problem as before, but with:

$$pq = \sqrt{2\pi\alpha} + hq$$

Plot .... on the y-axis and .... on the x-axis.

Find α as: .... and the uncertainty in α as: ....

Find h as: .... and the uncertainty in h as: ....

9. The same problem as before, but with:

$$p = \sqrt{\frac{3\alpha^2}{q} + 2h^3}$$

Plot .... on the y-axis and .... on the x-axis.

Find α as: .... and the uncertainty in α as: ....

Find h as: .... and the uncertainty in h as: ....

10. Two quantities, a frequency f and an angle θ, are related according to the relation

$$f = \sqrt{q \sin\theta} \tag{5.6}$$

where q is a parameter. The table below represents the result of a set of measurements of f and θ.

f [Hz]	θ [degrees]
1	0
2	8
3	13
4	30
5	50
6	90

(a) Find the value of q from a hand-made graph. Give the result in SI units.

(b) Suppose that the error in the slope is 10%. What is the error on the value of q calculated before? Give the result in SI units.

(c) Find the value of q from a computer-made graph, including the error (the error in the slope can be found using excel). Give the result in SI units

11. The relation between two physical quantities, T and L, is

$$T = \alpha L + \frac{2\pi}{q} \tag{5.7}$$

T [s]	L [m]
1	1
2	5
3	10.2
4	15
5	18.2
6	22.4
7	26.8
8	36
9	41
10	50

(a) Use the data above to find α (in SI units), including the error, from a linear plot.

(b) Use the data above to find q (in SI units), including the error, from a linear plot.

Solution: plotting T vs. L we expect to find a line with slope equal to a and intercept equal to $\frac{2\pi}{q}$. The best fit line has equation

$$y = (0.1878 \pm 0.0093)x + (1.2623 \pm 0.253) \tag{5.8}$$

from which

$$a = (0.188 \pm 0.009)\,\mathrm{s/m} \quad \text{and} \quad q = (5.0 \pm 1.0)\,\mathrm{s}^{-1}\,. \tag{5.9}$$

12. The relation between two physical quantities, T and L, is

$$T = \sqrt{aL + 2q} \tag{5.10}$$

T [s]	L [m]
0.5	2
2	3
3	4
4	6
5	7
6	8
6.5	9
8	13
9	15
10	17

(a) Use the data above to find a (in SI units), including the error, from a linear plot.

(b) Use the data above to find q (in SI units), including the error, from a linear plot.

Solution: plotting T^2 vs. L we expect to find a line with slope equal to a and intercept equal to $2q$. The best fit line has equation

$$y = (6.6148477 \pm 0.253499)x + (-17.8147208 \pm 2.460378016) \tag{5.11}$$

from which

$$a = (6.61 \pm 0.25)\,\mathrm{s}^2/\mathrm{m} \quad \text{and} \quad q = (-8.9 \pm 1.2)\,\mathrm{s}^2\,. \tag{5.12}$$

13. Two physical quantities, T and x, are related as

$$T = \frac{\pi}{\alpha}\sqrt{\frac{x}{2}} \tag{5.13}$$

Find the value of α, including the error, from the data below. Write the result in SI units.

x [m^2]	T [s]
1	0.41
1.96	0.61
4.41	0.82
9.61	1
10.89	1.12
16.81	1.14
24.01	1.30
33.64	1.40
44.89	1.45
77.44	1.65

14. Suppose that the relation in the problem above were

$$T = \frac{\pi}{\alpha}\sqrt{\frac{x+c}{2}} \tag{5.14}$$

Find α and c, including the error, from the data above.

Solution: If you square the expression you find

$$T^2 = \left(\frac{\pi^2}{2\alpha^2}\right) x + \left(\frac{\pi^2 c}{2\alpha^2}\right) . \tag{5.15}$$

This means that if we plot T^2 on the $y-$axis and x on the $x-$axis, we find a straight line with slope $m = \left(\frac{\pi^2}{2\alpha^2}\right)$ and intercept $b = \left(\frac{\pi^2 c}{2\alpha^2}\right)$. Therefore,

$$\begin{aligned} \alpha &= \sqrt{\frac{\pi^2}{2m}}, \qquad \delta\alpha = \alpha\left(\frac{1}{2}\frac{\delta m}{m}\right) \\ c &= \frac{2\alpha^2\, b}{\pi^2}, \qquad \delta c = c\left(2\frac{\delta\alpha}{\alpha} + \frac{\delta b}{b}\right) . \end{aligned} \tag{5.16}$$

Excel gives the equation of the line:

$$y = (0.031443462 \pm 0.004236863)x + (0.617751 \pm 0.135566) . \tag{5.17}$$

Therefore,

$$\begin{aligned} \alpha &= (12.5 \pm 0.8)\,\mathrm{m/s} \\ c &= (19.7 \pm 7.0)\,\mathrm{m}^2 . \end{aligned} \tag{5.18}$$

6 Problems on Basic Kinematics and Dynamics

6.1 Kinematics

1. I measure the velocity of an object which travels at constant speed along a straight line. The initial position of the object is (0.0 ± 0.1) cm, and the final position (100.0 ± 0.1) cm. The time taken to travel that distance is $t = (37 \pm 1)$ s. Write the velocity of the object, including the error, in SI units.

   Answer: $d = 100$cm, $\Delta d = 0.2$ cm, $t = 37$s, $\Delta t = 1$s, $v = 2.7027$ cm/s, $\delta v/v = 0.028027$, $\delta v = 0.0757487 \simeq 0.08$ cm/s, and so $v = (2.70 \pm 0.08)$ cm, which in SI units is $v = (2.70 \pm 0.08) \times 10^{-2}$ m.

6.2 Acceleration due to gravity

2. You can find the acceleration due to gravity, g, by dropping a stone to the ground from some height h, and measuring the time it takes to reach the ground. The relation between time and distance is

$$h = 1/2gt^2 \tag{6.1}$$

   Suppose that you drop the stone from 2.4 m, and it takes 0.8 s to reach the ground. Your uncertainty on the distance is 10 cm, and on the time is 0.1 s.

   (a) What is your value for the acceleration due to gravity? (include the error)

   (b) Are you compatible with the value $g = 9.81\,\mathrm{m/s^2}$?

3. In an experiment to find the acceleration due to gravity, g, you measure the period T of a pendulum for several different lengths L.

   Your data table is

L[m]	T[s]
0.1	0.69
0.3	1.10
0.5	1.40
0.7	1.82
1.0	1.93

   Remember that the period and the length are related by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

   (a) Use the above data to find g from a hand-made best fitting line. Neglect the uncertainties.

   (b) Use excel or graphical analysis to find g from the best fitting line. Include the uncertainties.

4. In a conical pendulum experiment you plot T (in SI units) versus $\sqrt{L}$ (in SI units), and you find the straight line

$$y = (0.90 \pm 0.02)\,x$$

Your value for the ratio of the masses is $\mu = 0.21 \pm 0.01$.

Remember that:

$$T = 2\pi\sqrt{\frac{L\mu}{g}}$$

Calculate g, including the error (in SI units).

Answer: $g = (10.2 \pm 0.9)\,\mathrm{m/s^2}$.

6.3 Newton's law

5. Consider an Atwood's machine system. The net force acting on the system is

$$F = (m_1 - m_2)g\,. \tag{6.2}$$

Suppose you use the two masses $m_1 = (135.0 \pm 0.2)\,\mathrm{g}$ and $m_2 = (102.2 \pm 0.2)\,\mathrm{g}$, and for the acceleration due to gravity you use $g = (9.8 \pm 0.1)\mathrm{m/s^2}$.

(a) Calculate F in Newton, including the error.

(b) Use the value of the force found to calculate the acceleration

$$a = \frac{F}{m_1 + m_2}\,. \tag{6.3}$$

Express the result, including the error, in SI units.

6. In an Atwood's machine setup, the acceleration of the system is given by

$$a = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\,, \tag{6.4}$$

where $m_1 = (110.5 \pm 0.2)\mathrm{g}$ and $m_2 = (72.3 \pm 0.2)\mathrm{g}$ are the masses of the two blocks, and $g = (9.82 \pm 0.01)\mathrm{m/s^2}$ is the acceleration due to gravity.

Calculate the acceleration including the uncertainty.

7. In the Atwood's machine experiment the net force is

$$F = (m_1 - m_2)\,g$$

and the acceleration is

$$a = \frac{F}{m_1 + m_2}$$

You collect data, and then plot the acceleration (in $\mathrm{m/s^2}$) on the y-axis and the net force (in N) on the x-axis. The software that you use (excel or graphical analysis) to analyze the data gives for the equation of the best fitting line:

$$y = (4.9233671 \pm 0.0037690)x - (0.002621 \pm 0.000033)$$

(a) Find the total mass (in grams) of the system, including the error.

(b) What is the minimal force necessary to overcome the static friction of the pulleys?

6.4 Linear momentum

8. In an experiment to study the conservation of linear momentum, you collide two masses m_1=38 g and m_2=51 g . The uncertainty on the masses is negligible. The speeds of the colliding particles are $v_1 = (3.1 \pm 0.1)$ m/s and $v_2 = (2.4 \pm 0.1)$ m/s. After the collision the two objects stick together and move with the speed $v_f = (2.8. \pm 0.2)$ m/s.

   (a) Calculate the initial and final momentum, $p_i = m_1v_1+m_2v_2$,and $p_f = (m_1+m_2)v_f$,including the error.

   (b) Is the momentum is conserved ($p_i = p_f$) within the error?

9. In the ballistic pendulum experiment, you can find for the velocity of the bullet

$$v = \sqrt{\frac{gx^2}{2H}}$$

   where $H = (68.5 \pm 0.1)$ cm is the height of the table, $g = 9.81$ is the acceleration due to gravity (known with negligible error), and $x = (284 \pm 2)$ cm is the distance where the bullet lands, and for the velocity of the bullet+pendulum system

$$V = \sqrt{2gh}$$

   where $h = (17 \pm 1)$ cm. The mass of the pendulum is $M = 148$ g and the mass of the bullet is $m = 50$ g. The masses are known with negligible error.

   (a) Calculate $P_i = mv$, including the error.

   (b) Calculate $P_f = (M + m)V$, including the error.

   (c) Is the momentum of the system conserved within the error?

6.5 Rigid bodies

10. In an experiment to study Newton's laws applied to a rigid body, you apply the force $F_1 = (0.4 \pm 0.05)$N, to a body, with a lever arm $d_1 = (14.4 \pm 0.2)$cm, and a force F_2 at a distance R from the rotation center. You vary F_2 and the angle θ between F_2 and R. Equilibrium results if

$$F_1d_1 = F_2R\sin\theta \tag{6.5}$$

    If you plot F_2 vs. $(R\sin\theta)^{-1}$ (both in SI units) you find the curve

$$y = (0.07 \pm 0.01)x + (0.01 \pm 0.005)$$

    (a) Calculate the force F_1 from the graph (including the error).

    (b) Do you confirm the equation for static equilibrium (6.5) within the error?

11. Consider the static configuration of a cylinder of mass $m = (120 \pm 1)$g, attached to a spring with constant k, and partially immersed in a fluid. The total force acting on the mass is

$$F_{tot} = mg - kx - \rho g V_{sub} = 0 \tag{6.6}$$

    where $g = (9.80 \pm 0.02)\text{m/s}^2$, $\rho = (1075 \pm 5)\text{kg/m}^3$, and V_{sub} is the volume of the cylinder immersed in water.

To find k you measure the spring stretch from the equilibrium position, x, for different values of V_{sub}, and find the best fit to your data (in SI units)

$$y = (-2742 \pm 2)x + (0.32 \pm 0.01)$$

(a) Calculate $k \pm \Delta k$ from the slope of the graph.

(b) Calculate $k \pm \Delta k$ from the intercept of the graph.

(c) Do the two values found agree within the error?

7 Problems on Waves and Sound

1. In an experiment to study standing waves, you use a string whose mass per length is $\mu = (1.8 \pm 0.1) \times 10^{-3}$kg/m. You look at the fundamental mode, whose frequency f is related to the length L and tension T of the string by the following equation

$$L = \frac{1}{2f}\sqrt{\frac{T}{\mu}}\,. \tag{7.1}$$

   You make a plot with L on the y-axis and $\sqrt{T}$ on the x-axis, and find that the best fitting line is

$$y = (8.3 \pm 0.3) \times 10^{-3}\, x + (0.2 \pm 0.4).$$

   in SI units. What is the value of the frequency of the wave (including the error)? Express your result in SI units (Hz).

2. The temperature in a room is $T = 19.8$ degrees Celsius (**the uncertainty in the temperature is negligible**). The relation that describes the dependence of the velocity of the sound waves in air on the temperature is

$$v = v_0 \times \sqrt{1 + \frac{T}{273}} \tag{7.2}$$

   where $v_0 = (330 \pm 2)$ m/s is the velocity of the sound in air (at zero Celsius) and T is the temperature of the air in degrees Celsius.

   (a) Calculate the velocity of the sound waves v, including the error. Express your result in SI units.

   (b) **More challenging question:** Suppose now that the uncertainty in the temperature is $\Delta T = 0.2$ degrees Celsius. What would the uncertainty in the velocity be in this case?

3. In order to study the dependence of the velocity of the sound waves in air on the temperature, you collect data of the velocity at different temperatures. The relation that describes this dependence is

$$v = v_0 \times \sqrt{1 + \frac{T}{T_0}} \tag{7.3}$$

   If you plot v^2 (in $\mathrm{m}^2/\mathrm{s}^2$) on the $y-$axis, and T (in C) on the $x-$axis, you find (from the least-square-fit analysis) the relation

$$y = (398 \pm 8)\mathrm{m}^2/(\mathrm{s}^2\,\mathrm{C}) + (108100 \pm 1000)\mathrm{m}^2/\mathrm{s}^2 \tag{7.4}$$

   Calculate v_0 and T_0 including the errors.

8 Problems on Thermodynamics

1. A piece of metal with mass $m_m = (312 \pm 2)$ g, initial temperature $T_m = (21 \pm 1)$ C, and specific heat $c_m = (128 \pm 1)$ J/kgC, is brought into contact with water of mass $m_w = (1.472 \pm 2)$ g, and initial temperature $T_w = (88 \pm 1)$ C. The specific heat of the water is $c_w = (4186 \pm 1)$ J/kgC.

   To find the equilibrium temperature T_f, (neglecting the heat exchange with the environment) you need to impose the equation

$$m_m c_m (T_f - T_m) + m_w c_w (T_f - T_w) = 0\,. \tag{8.1}$$

   Calculate the equilibrium temperature (T_f) of the water+metal system, including the error.

2. A piece of metal with mass $m_m = 308$ g, initially at room temperature $T_m = (21.0 \pm 0.2)$ C, is brought into contact with water of mass $m_w = 192$ g, and initial temperature $T_w = (82 \pm 0.8)$ C. The equilibrium temperature is $T_f = (72 \pm 1)$C. The specific heat of the water is $c_w = (4180 \pm 10)$ J/kgC.

   To find the specific heat of the metal, c_m, you use the equation

$$m_m c_m (T_f - T_m) + m_w c_w (T_f - T_w) = 0\,. \tag{8.2}$$

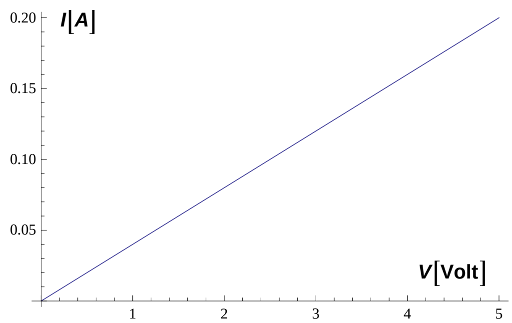
   Calculate the the specific heat of the metal, c_m, including the error.

9 Problems on DC-Circuits

9.1 Ohm's law

1. In a DC circuit, the electric current is $I = (15.8 \pm 0.1)\mu$A, and the potential is $V = (2.32 \pm 0.02)$V. Find the resistance including the error. Write the result in SI units, and round properly (keep only one or two significant digits for the error, and round the result so that its accuracy is the same as the accuracy of the error).

2. The figure below shows the current vs. potential graph for a resistor.



   (a) Find the resistance;
   Answer: Slope=$(1/25)\Omega^{-1} = 0.04\Omega^{-1}$, so $R = 25\Omega$.

   (b) Suppose that the error in the slope s of the graph is $\delta s = 1.17647 \times 10^{-3}\Omega^{-1}$. Calculate the error δR in the resistance.
   Answer: $\delta s/s = \delta R/R$, so $\delta R/R = 0.0294118$.
   So $\delta R = R \times 0.0294118 = 0.735294\Omega \simeq 0.7\Omega$.

   (c) Write the result for the resistance (including the error) properly rounded.
   Answer: $R = (25.0 \pm 0.7)\Omega$

3. The best fit line for a current vs. potential graph for a resistor has equation (we are using SI units)

$$I = (3.08 \pm 0.02) \times 10^{-3}\, V + (0.002 \pm 0.004)$$

   (a) Find the resistance.

   (b) Calculate the error δR in the resistance.

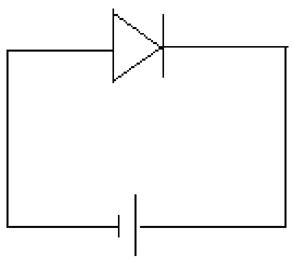
   (c) Write the result for the resistance (including the error) properly rounded.

9.2 Diodes

4. A diode is connected *in reverse bias* to a 1.5 V battery. What is (approximately) the current that you expect it would flow through it?

5. Draw the Current versus Voltage characteristic (CVVC) of the semiconductor diode

6. Consider the circuit below



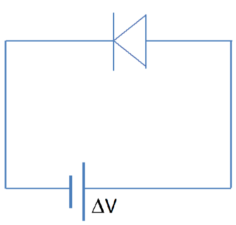
   (a) Do you expect a linear relation between the voltage and the current in the circuit? Explain.

   (b) Indicate the **p** and **n** part of the diode in the figure

   (c) Is the **p-n** junction in forward or reverse bias?

   (d) What is the current that you expect to measure in this configuration (positive, negative, zero, directly proportional to the potential, inversely proportional to the potential)?

7. Consider the circuit below



   (a) Do you expect a linear relation between the voltage and the current in the circuit?

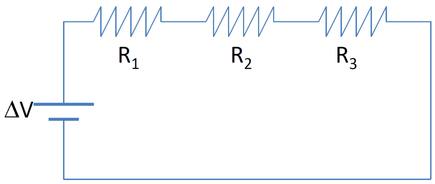
   (b) Indicate the **p** and **n** part of the diode in the figure

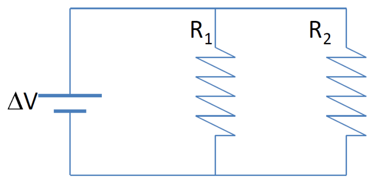
   (c) Is the **p-n** junction in forward or reverse bias?

   (d) What is the current that you expect to measure in this configuration (zero, directly proportional to the potential, inversely proportional to the potential, exponentially related to the potential, logarithmically relater to the potential)?

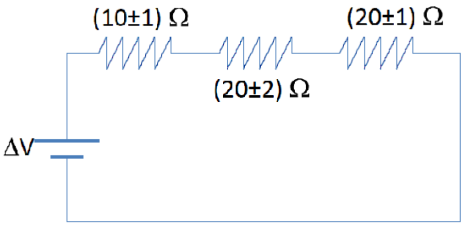
9.3 Parallel and series of resistors and Kirchhoff's rules

8. Find the formula for equivalent resistance and its uncertainty for the circuits shown below

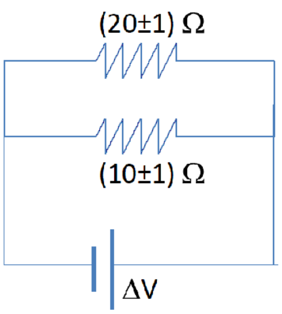




9. Find the equivalent resistance (including the error) of the circuit shown below



10. Find the equivalent resistance (including the error) of the circuit shown below



11. Consider the circuit in figure 1. The values of the resistances are $R_1 = (22.6 \pm 0.2) \times 10^3\Omega$, $R_2 = (8.6 \pm 0.1) \times 10^3\Omega$, $R_3 = (9.0 \pm 0.1) \times 10^3\Omega$.

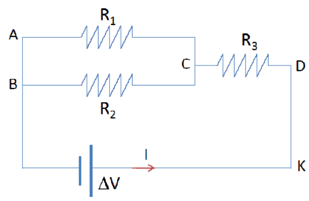


Figure 1: Figure of circuit for problems 11 and 12

(a) Draw the equivalent circuit (with just one resistor, R_{eq});

(b) Calculate R_{12}, including the error. Write $R_{12} \pm \delta R_{12}$ properly rounded.

(c) Calculate R_{eq}, including the error. Write $R_{\text{eq}} \pm \delta R_{\text{eq}}$ properly rounded.

(d) Suppose that $\Delta V = (2.00 \pm 0.02)$V. Find the current I including the error.

12. Consider the circuit in figure 1. Circle the correct statements.

(a) The potential in A is the same as the potential in B.

(b) The potential difference between A and D is zero.

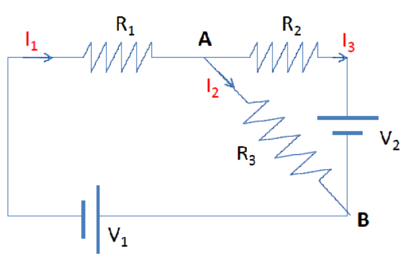
(c) The potential difference between C and D is ΔV.

(d) The potential difference between D and K is ΔV.

(e) The potential difference between A and K is ΔV.

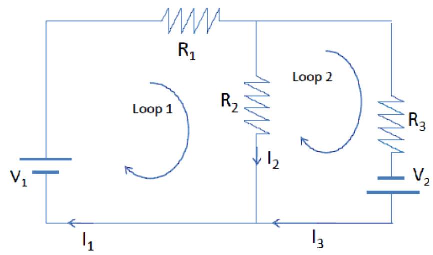
(f) The current flowing through R_1 is the same as the current flowing through R_3.

13. Consider the circuit in the figure below with $R_1 = (15 \pm 1.5)\text{k}\Omega$, $R_2 = (10 \pm 1)\text{k}\Omega$, $R_3 = (50 \pm 5)\text{k}\Omega$, $V_1 = (5.0 \pm 0.1)$ V, $V_2 = (10.0 \pm 0.1)$ V. The currents are known with negligible uncertainties: $I_1 = 2.9 \times 10^{-4}$A, $I_2 = 7.1 \times 10^{-5}$A, $I_3 = 2.1 \times 10^{-4}$A.



Verify the Kirchhoff's equations for loop 1 and loop 2, and specify if they are satisfied within the error. (Show the calculations)

14. Consider the circuit in the figure below with $R_1 = (15 \pm 1.5)\text{k}\Omega$, $R_2 = (10 \pm 1)\text{k}\Omega$, $R_3 = (50 \pm 5)\text{k}\Omega$, $V_1 = (5.0 \pm 0.1)$ V, $V_2 = (10.0 \pm 0.1)$ V. The currents are known with negligible uncertainties: $I_1 = 2.9 \times 10^{-4}$A, $I_2 = 7.1 \times 10^{-5}$A, $I_3 = 2.1 \times 10^{-4}$A.



Verify the Kirchhoff's equations for loop 1 and loop 2, and specify if they are satisfied within the error. (Show the calculations)

9.4 RC-circuits

15. In a RC circuit the capacitor has capacitance $C = (15 \pm 1)\mu$F, the resistance is $R = (2500 \pm 200)\Omega$, and the potential of the battery is $V = 15$V.

    (a) Find the time constant of the circuit (including the error).
    Solution: The time constant of a RC circuit, which means a circuit with a resistor and a capacitor (besides, of course, a generator), is

$$\tau = RC\,. \tag{9.1}$$

    Notice that 1 Farad times 1 Ohm equals 1 second. In our case,

$$\tau = 2500 \times 15 \times 10^{-6} s \left[1 \pm \left(\frac{2}{25} + \frac{1}{15}\right)\right] = (3.75 \pm 0.55) \times 10^{-2}\,\text{s}\,. \tag{9.2}$$

    (b) What is the charge on the capacitor after 100μs? (disregard the error).
    Hint: Remember that

$$Q(t) = CV\left(1 - e^{-t/\tau}\right) \tag{9.3}$$

    (c) What is the current in the circuit after 100μs? (disregard the error).
    Hint: Remember that

$$I(t) = \frac{V}{R}e^{-t/\tau} \tag{9.4}$$

16. In a RC circuit the capacitor has capacitance $C = (1.5 \pm 0.1)\mu$F, the resistance is $R = (1200 \pm 200)\Omega$, and the potential of the battery is $V = 11$V.

    (a) Find the time constant of the circuit (including the error).
    (b) What is the charge on the capacitor after 20μs? (disregard the error).
    (c) What is the current in the circuit after 20μs? (disregard the error).
    (d) After how long is the current in the circuit 20% of the initial current? (disregard the error).

(e) After how long is the charge on the capacitor 20% of the final charge? (disregard the error).

17. In a RC circuit the capacitor has capacitance $C = (11 \pm 1)\mu$F, and there are 2 resistors in series with resistances $R_1 = (1500 \pm 200)\Omega$ and $R_2 = (500 \pm 100)\Omega$. The potential of the battery is $V = (12 \pm 1)$V.

    (a) Find the equivalent resistance of the circuit (including the error).
    Answer: $R_{eq} = (2000 \pm 300)\,\Omega$

    (b) Find the time constant of the circuit (including the error).
    Answer: $\tau = R_{eq}C = (2000 \times 11 \times 10^{-6})\text{s} = (2.2 \pm 0.5) \times 10^{-2}\text{s}$

    (c) What is the initial current in the circuit (including the error).
    Answer: $I = \frac{V}{R_{eq}} = (6.0 \pm 1.4) \times 10^{-3}\text{A}$

    (d) What is the current in the circuit after 10^{-2}s (disregard the error).
    Answer: 3.80842×10^{-3}A

18. During the charging process of a capacitor in a RC-circuit, the potential across the capacitor evolves as

$$V_c = V\left(1 - e^{-t/\tau}\right) \tag{9.5}$$

You collect data of the voltage across the capacitor at different times. Your best fit is

$$y = A\left(1 - e^{-Bt}\right) \tag{9.6}$$

with $A = (1.47 \pm 0.001)$ in SI units, and $B = (152 \pm 2)$ in SI units.

    (a) Find the time constant of the circuit (including the error).

    (b) What is the meaning of the parameter A?

19. During the charging of a capacitor in a RC-circuit, the current in the circuit evolves as

$$I = \frac{V}{R}e^{-t/\tau} \tag{9.7}$$

with $V = 12$ V (negligible uncertainty). You collect data of the current in the circuit at different times, and plot $\ln I$ versus t. Your best fit is

$$y = A + Bx \tag{9.8}$$

with $A = -4.6$ (negligible uncertainty), and $B = -25.6 \pm 0.1$.

    (a) Find the time constant of the circuit (including the error).

    (b) Find the resistance R of the circuit (ignore the error)?

10 Problems on Resonant Circuits

1. What is the resonant frequency in an LRC circuit?

2. How does the resonant frequency in an LRC circuit change is the capacitance C is doubled?

3. How does the resonant frequency in an LRC circuit change is the resistance R is doubled?

4. What is the definition of the quality factor Q in an LRC circuit?

5. The value of the components in a LRC circuit is $R = (11 \pm 2) \times 10^3\,\Omega$, $L = (10 \pm 1)\,\mathrm{mH}$, $C = (5 \pm 1)\mu\mathrm{F}$. The maximal voltage of the generator is $V = (10 \pm 1)\,\mathrm{V}$.

   (a) Calculate the resonant frequency, including the error.

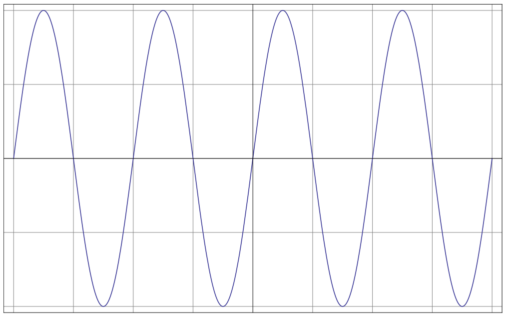
   (b) Calculate the maximal current in the circuit, including the error. (NOTE: The current is maximal when the frequency is resonant. In this case $I_{max} = V_{max}/R$, since the effects of L and C cancel each other).

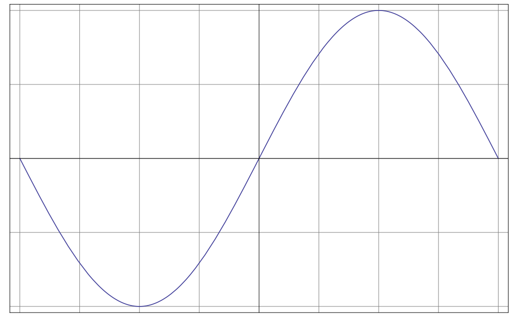
   (c) Calculate the quality factor Q, including the error, using the formula

$$\frac{1}{Q} = R\sqrt{\frac{C}{L}} \tag{10.1}$$

11 Problems on the Oscilloscope

1. An oscilloscope has the TD knob set to 0.5ms, and the VD knob set to 2.5V.



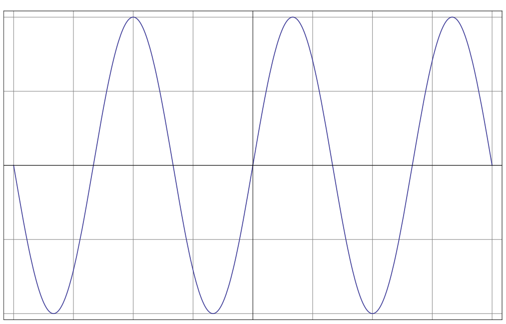


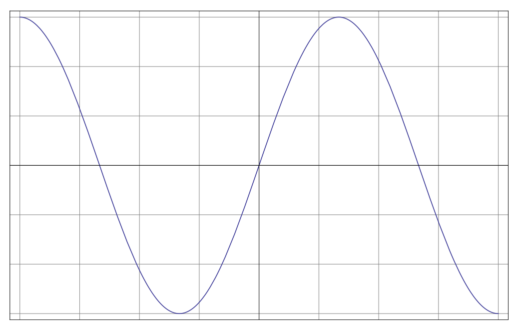
   (a) Find amplitude, period and frequency of the signal shown in the figure above (left panel)

   (b) Find amplitude, period and frequency of the signals shown in the figure above (right panel)

   IMPORTANT NOTE: Remember that the amplitude of a signal is the height from the middle to the top, not from the bottom to the top. For example, the amplitude of the signal in the above problem is 2 divisions (not 4 divisions), which means 5 V.

2. An oscilloscope has the TD knob set to 2ms, and the VD knob set to 5V.





   (a) Find amplitude, period and frequency of the signal shown in the figure above (left panel)

   (b) Find amplitude, period and frequency of the signals shown in the figure above (right panel)

12 Problems on Magnetic Field

1. What is the magnitude of the magnetic force exert on a straight wire with current I and length L, if the magnetic field lines (of intensity B) form an angle θ with the wire?

2. The direction of the magnetic force is *1)* parallel to the wire, *2)* perpendicular to the wire or *3)* at an angle θ with respect to the wire?

3. What is the magnitude of the magnetic force exert on a straight wire with current $I = (5 \pm 0.5)$A and length $L = (5.0 \pm 0.1)$cm, immersed in a magnetic field of intensity $B = (0.60 \pm 0.03)$T, if the magnetic field lines form an angle $\theta = \frac{\pi}{6}$ with the wire? (ignore the error in the angle).

4. The force exert on a wire long L and with a current I by a magnetic field perpendicular to the wire is $|\vec{F}| = IL|\vec{B}|$. An experimental plot shows $|\vec{F}|$ as a function of L. The plot is a straight line, with slope $s = (10 \pm 1) \times 10^{-5}$A·T. The current in the wire is $I = (15 \pm 1)$mA. Find $|\vec{B}|$, including the error (the SI units for B is tesla, T).

   Answer:

$$B = \frac{s}{I} = 6.67 \times 10^{-3} T \tag{12.1}$$

$$\delta B = B \left(\frac{\delta s}{s} + \frac{\delta I}{I} \right) = 6.67 \times 10^{-3} T \times 0.1667 = 1.1 \times 10^{-3} T \tag{12.2}$$

13 Problems on Electromagnetic Waves and Optics

1. A beam of light strikes the boundary between two media (from medium-1 to medium-2) with an incident angle $\theta_i = 25°$. The angle of refraction is $\theta_t = 15°$. These angles are known with negligible uncertainty. The first medium has a refractive index $n_1 = 1.2 \pm 0.1$. Calculate the index of refraction of the second medium, including the error.

2. A beam of light strikes the interface between air and glass, and is refracted in the glass. The angle of refraction θ_t is related to the incident angle θ_i by the Snell's relation

$$n_{\text{air}} \sin\theta_i = n_{\text{glass}} \sin\theta_t \,, \tag{13.1}$$

with $n_{\text{air}} = 1$ (with negligible uncertainty). You plot $\sin\theta_t$ (on the y-axis), vs. $\sin\theta_i$ (on the x-axis), and find that the best fit line has equation

$$y = (7.23 \pm 0.05) \times 10^{-1} x + (0.022 \pm 0.024). \tag{13.2}$$

Calculate the index of refraction of the glass, including the error.

Answer: $n_{\text{glass}} = (1.38 \pm 0.01)$.

3. A convex lens has a focal length of 5.0 cm. A concave lens has a focal length of -15.0 cm. A 1cm high object is placed 25.0 cm to the left of the convex lens. The concave lens is placed 10.0 cm to the right of the convex lens.

A) How far is the image from the object?

B) How high is the image?

C) Is the image upright or inverted?

Solution: It is recommended that you make a drawing.

From the problem, d_{o1}=25cm, so the primary image is located at

$$d_{i1} = \left(\frac{1}{f_1} - \frac{1}{d_{o1}}\right)^{-1} = \frac{25}{4}\,\text{cm} = 6.25\,\text{cm} \tag{13.3}$$

from the first lens. Since this value is positive, the image is to the right of the first lens.

The magnification of the primary image is

$$m_1 = -\frac{d_{i1}}{d_{o1}} = -\frac{1}{4}\,. \tag{13.4}$$

The primary image is the object for lens 2. From your drawing and the data, you can see that it has to be located 3.75 cm to the left of lens 2. Therefore, d_{o2}=3.75 cm. So the image from lens 2 is located at

$$d_{i2} = \left(\frac{1}{f_2} - \frac{1}{d_{o2}}\right)^{-1} = -3\,\text{cm} \tag{13.5}$$

Since this value is negative, the image is to the left of the second lens.

Therefore, the distance of the final image to the object is 25cm+10cm-3cm=32cm.

The magnification of the secondary (final) image, with respect to the primary one, is

$$m_2 = -\frac{d_{i2}}{d_{o2}} = \frac{4}{5}\,. \tag{13.6}$$

So, the total magnification is

$$m_1 m_2 = -\frac{1}{5}\,. \tag{13.7}$$

Notice that this is negative, so the image is inverted.

Finally, the height of the image is

$$h_i = -\frac{1}{5} h_o = -2\,\mathrm{mm}\,. \tag{13.8}$$

4. The intensity of light from a light bulb is related to the distance from the source according to the relation

$$I = \frac{P}{4\pi\, R^2} \tag{13.9}$$

You want to measure the power from a plot of the intensity (I) versus the inverse distance squared R^{-2}. Your best fit lines is

$$y = (36 \pm 2)x + (0.23 \pm 0.65) \tag{13.10}$$

in SI units (remember that the SI units for power is W). What is the power of the bulb (in Watts), including the error?

5. A monochromatic beam of light of wavelength 600 nm is incident normally on a diffraction grating with a slit spacing of 1.70×10^{-4} cm. What is the angle for the first order maximum (above the central bright fringe)?

6. A monochromatic beam of light of wavelength 533 nm is incident normally on a diffraction grating with a slit spacing of 1.20×10^{-4} cm. What is the angle for the first order maximum (above the central bright fringe)?